

# Chiral Thirring-Wess Model

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The vector type of interaction of the Thirring-Wess model was replaced by the chiral type and a new model was presented which was termed as chiral Thirring-Wess model in [8]. The model was studied there with a Faddeevian class of regularization. Few ambiguity parameters were allowed there with the apprehension that unitarity might be threatened like the chiral generation of the Schwinger model. In the present work it has been shown that no counter term containing the regularization ambiguity is needed for this model to be physically sensible. So the chiral Thirring-Wess model is studied here without the presence of any ambiguity parameter and it has been found that the model not only remains exactly solvable but also does not lose the unitarity like the chiral generation of the Schwinger model. The phase space structure and the theoretical spectrum of this new model have been determined in the present scenario. The theoretical spectrum is found to contain a massive boson with ambiguity free mass and a massless boson.

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## I. INTRODUCTION

In lower dimensional field theory Thirring and Wess jointly presented an interesting and exactly solvable model [1] few years after the presentation of the Schwinger model [2]. The nature of interaction between the matter and gauge field in these two models were identical and both the models were capable of describing generation of mass and had been studied over the years not only because of their surprising exactly solvable nature but also for their ability to describe mass generation, confinement as well as de-confinement aspect of fermions etc., [1–8]. In the Schwinger model it was found that photon acquired mass via a kind of dynamical symmetry breaking keeping the gauge symmetry of the model intact. Recently, it was shown by us that a novel back ground interaction emerged out when an attempt was made to study this model in the non commuting space time setting [9–11].

Though the joint analysis of Thirring and Wess [1] revealed that mass generation of photon can also be explained by the model coined by themselves nevertheless gauge symmetric structure was not possible to maintain in this model to start with [1]. This model was named after Thirring and Wess and it was known as Thirring-Wess model. Recently, an attempt has been made in [12], for systematic functional integral bosonization of this mode.

After few years of presentation of the Thirring-Wess model, chiral generation of Schwinger model was perused in [13], however the model remained less attractive over a long period of time because of its inability to provide a unitary field theoretical description. But it attracted huge attentions and gradually acquired a significant position in lower dimensional field theory after the removal of the non-unitary problem by Jackiw and Rajaraman [14] taking into account anomaly into consideration. Anomaly played interesting as well as surprising role for this model to place it in a well acceptable position. The welcome entry of the anomaly and the suitable exploitation of the ambiguity involved therein made Jackiw-Rajaraman version of Chiral Schwinger model [14–19] along with the other independent regularized version of that model [20–23] so popular in lower dimensional field theory regime.

Like the Schwinger model the Thirring-Wess model also describes an interacting theory of massless fermion with Abelian gauge field in two dimension and the nature of interaction in both the models are identical ( vector like). The difference lies only in the gauge field sector. In the Schwinger model the kinematics of the gauge field enters through the Maxwell type lagrangian, however the Thirring-Wess model can be considered as a study of QED, viz., Schwinger model [2, 3] replacing Maxwell's field by Proca field and that very replacement breaks the gauge symmetry of the model at the classical level. However, no constraint related to inconsistency stands as a hindrance against the way of Thirring-Wess model [1] to mark it as a sound field theoretical model.

It is true that the so called non-confining Schwinger model [24–26] is a structurally equivalent gauge non-invariant model to the Thirring-Wess mode, but there lies a crucial difference between these two. In the Thirring-Wess mode the masslike term for the gauge field entered at the classical level however in the so called non-confining Schwinger model the same type of masslike term gets involved through the one loop correction which contains an ambiguity parameter like Jackiw-Rajaraman version of Chiral Schwinger model.

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An attempt was made in [8] in order to have the chiral generation of the Thirring-Wess mode in the similar way the chiral generation was made for the Schwinger model in [13]. The chiral generated version however had to be studied there with a Faddeevian class of regularization and few ambiguity parameter were allowed to enter during the process of removal of divergence of the fermionic determinant in course of going over to its bosonized version with an apprehension that it might suffer from the unwanted non unitarity problem like the chiral generation of the Schwinger model [13].

Let us now explore a bit related to the necessity of the allowance of ambiguity parameter in the QED and chiral QED. It is known that the vector Schwinger model remained unitary and exactly solvable in absence of any ambiguity parameter, particularly when its bosonized version is concerned, however the chiral generation of this model [13] faced a severe non-unitary problem. The Thirring-Wess model in its usual version was free from non-unitary problem and was exactly solvable even in the absence of any ambiguity parameter in its bosonized version too. A question, therefore, automatically generates whether the chiral generation of Thirring-Wess model can provide a physically sensible theory even in the absence of any ambiguity parameter like its ancestor where the basic interaction was vector like? The search for the answer to this question leads to an interesting investigations which we would like to explore in this paper. To investigate it in detail an attempt has been made here to investigate the chiral Thirring-Wess model without any counter term containing ambiguity parameter. One may however be free to allow ambiguity involved counter term in this situation too as long as it remains physically sensible. When the ambiguity involved one loop correction is allowed to enter that also suggests another direction of investigation. It suggests how the theoretical spectrum gets modified when this one loop correction is allowed. A special type of ambiguity involved counter term is allowed in our analysis which resembles the Jackiw-Rajaraman's counter term for the chiral Schwinger model and a pedagogical illustrations has been made to see the role of ambiguity parameter towards the modification of the theoretical spectrum.

The paper is organized as follows. Sec. II, contains the chiral generation of Thirring-Wess model. In Sec. III, the model has been analyzed using lagrangian formulation and free field solution is obtained. In Sec. IV the phase space structure of the mode is determined using Dirac method of quantization of constrained system. Sec. V. contains the concluding remarks.

## II. CHIRAL GENERATION OF THE THIRRING-WESS MODEL

The Chiral generation of the Thirring-Wess model was attempted in our previous work [8] by the following generating functional

$$Z[A] = \int d\psi d\bar{\psi} e^{\int d^2x \mathcal{L}_f} \quad (1)$$

with

$$\begin{aligned} \mathcal{L}_f &= \bar{\psi} \gamma^\mu [i\partial_\mu + e\sqrt{\pi} A_\mu (1 - \gamma_5)] \psi \\ &= \bar{\psi}_R \gamma^\mu i\partial_\mu \psi_R + \bar{\psi}_L \gamma^\mu (i\partial_\mu + 2e\sqrt{\pi} A_\mu) \psi_L. \end{aligned} \quad (2)$$

Here dynamics of the  $A_\mu$  field was governed by the Proca field and the lagrangian of which was given by

$$\mathcal{L}_{Praca} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu. \quad (3)$$

It is known that the vector and axial vector current that couple with the gauge field are defined by  $J_\mu = \bar{\psi} \gamma_\mu \psi$  and  $J_\mu^5 = \bar{\psi} \gamma_\mu \gamma_5 \psi$  respectively. In order to make chiral generation of the usual Thirring-Wess model we replaced the vector type of interaction  $\bar{\psi} \gamma_\mu \psi A^\mu$  by the chiral type  $\bar{\psi} \gamma_\mu (1 - \gamma_5) \psi A^\mu$ . It was mentioned there that the right handed fermion remained uncoupled for this type of chiral interaction whereas the left handed fermion got coupled with the gauge field. As a result, integration over this right handed part led to a field independent counter part which was absorbed within the normalization. However, the integration over the left handed fermion was not like that. The computation of the integration of the left handed part is much involved and plays crucial role to make the model sensible as well as physically acceptable in the lower dimensional field theoretical regime. After integrating out of the lefthanded part the generating functional in a general form can be written down as

$$\begin{aligned} Z[A] &= \int d\psi_L d\bar{\psi}_L \bar{\psi}_L \gamma^\mu (i\partial_\mu + 2e\sqrt{\pi} A_\mu) \psi_L \\ &= \exp \frac{ie^2}{2} \int d^2x A_\mu [M_{\mu\nu} - (\partial^\mu + \tilde{\partial}^\mu) \frac{1}{\square} (\partial^\nu + \tilde{\partial}^\nu)] A_\nu. \end{aligned} \quad (4)$$

Here  $M_{\mu\nu}$  contains the one loop correction effect needed to remove the divergence of the fermionic determinant. Some regularization ambiguity also enters there during the course of removing the divergence. In general  $M_{\mu\nu}$  can take any form however the maintenance of unitarity and Lorentz invariance put different constraints and allows some suitable structure. One admissible structure was considered in [8]. In the present context we are interested to study chiral generation of the Thirring-Wess model with out any ambiguity involved term to investigate whether it can provide any physically sensible theory. It is known that in case of the chiral generation of the Schwinger model [13] it was not possible to have a sensible theory without the help of any ambiguity parameter.

The above generating functional (4) can be expressed in terms of some auxiliary field and when it is done so in terms of the an auxiliary field  $\phi(x)$  it turns out to the following

$$Z[A] = \int d\phi e^{i \int d^2x \mathcal{L}_B}, \quad (5)$$

with

$$\mathcal{L}_B = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + e(g^{\mu\nu} - \epsilon^{\mu\nu})\partial_\nu \phi A_\mu + \frac{1}{2}e^2 M_{\mu\nu} A_\mu A^\mu. \quad (6)$$

Since we are motivated here to study the chiral generation of Thirring-Wess model with out any ambiguity parameter the effect of  $M_{\mu\nu}$  is ignored here and the total lagrangian density, i.e., the bosonized lagrangian density along with the Proca background now reads

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_B + \mathcal{L}_{Praca}, \\ &= \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + e(g^{\mu\nu} - \epsilon^{\mu\nu})\partial_\nu \phi A_\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu, \\ &= \frac{1}{2}(\dot{\phi}^2 - \phi'^2) + e(\dot{\phi} + \phi')(A_0 - A_1) + \frac{1}{2}(\dot{A}_1^2 - A_0'^2) + \frac{1}{2}m^2(A_0^2 - A_1^2). \end{aligned} \quad (7)$$

We have chosen  $\epsilon^{01} = +1$ . Let us recall that when this type of chiral generalization was attempted for the Schwinger model in [13], the model though did not loose its solvability nevertheless unitarity failed to be maintained and the model remained non attractive over a long period of time because of its inability to provide the necessary condition for being physically sensible and was left un studied till this severe non-unitary problem was removed in [14]. So a question may automatically be arise whether the chiral generalization of Thirring-Wess model face the same problem or it is free from that severe un-physical situation? To explore it let us begin our analysis in the following section on the model proposed here.

### III. DETERMINATION OF THEORETICAL SPECTRUM THROUGH LAGRANGIAN FORMULATION

A careful look reveals that the the electromagnetic current related to lagrangian density (7) that takes part in the interaction with matter field is

$$J^\mu = e(g^{\mu\nu} - \epsilon^{\mu\nu})\partial_\nu \phi + m^2 A^\mu. \quad (8)$$

Note that  $\partial_\mu J^\mu \neq 0$ . So unlike the Schwinger model the current does not conserve. Let us now proceed to study the model through the standard lagrangian formulation. The Euler-Lagrange equations for the fields  $A_\mu$  and  $\phi$  that flow from the same lagrangian density (7) are

$$\partial_\mu F^{\mu\nu} + e(g^{\mu\nu} - \epsilon^{\mu\nu})\partial_\nu \phi + m^2 A^\mu = 0, \quad (9)$$

$$\square \phi + e(g^{\mu\nu} - \epsilon^{\mu\nu})\partial_\nu A_\mu = 0. \quad (10)$$

Solving the above equations one may obtain the free field solutions corresponding to the model (7) to which we now turn. From equation (10) we can write

$$\phi = -e(g^{\mu\nu} - \epsilon^{\mu\nu})\frac{\partial_\mu A_\nu}{\square}. \quad (11)$$

With the use of equation (11), equation (9) turns into

$$\partial_\mu F^{\mu\nu} + e^2(g^{\nu\lambda} - \epsilon^{\nu\lambda})\frac{\partial_\lambda \partial_\kappa}{\square}(g^{\kappa\mu} - \epsilon^{\kappa\mu})A_\mu + m^2 A^\nu = 0. \quad (12)$$

It can be shown that the above equations (9) and (10) will be satisfied for the following most general form of the field  $A_\mu$ :

$$A^\mu = -\frac{e}{m^2}(\partial^\mu \phi + \frac{(e^2 - m^2)}{e^2} \epsilon^{\mu\nu} \partial_\nu \phi - \frac{m^2}{e^2} \epsilon^{\mu\nu} \partial_\nu \chi), \quad (13)$$

Let us see how does it take place? From equation (13) it can be shown that

$$g^{\mu\nu} \partial_\mu A_\nu = -\frac{e}{m^2} \square \phi, \quad (14)$$

and

$$\epsilon^{\mu\nu} \partial_\mu A_\nu = -\frac{e}{m^2} [\frac{(e^2 - m^2)}{e^2} \square \phi - \frac{m^2}{e^2} \square \chi], \quad (15)$$

We, therefore, find that equations (14) and (15) are capable of reproducing the equation (10) as well as equation (11), if the field  $\chi$  satisfies equation  $\square \chi = 0$ . Using the expression of  $A_\mu$  given in (13) a straightforward calculation shows that  $A_\mu$  and  $\phi$  are related by the following second order differential equations.

$$\square A^\mu = -\frac{e}{m^2}(\partial^\mu \square \phi + \frac{(e^2 - m^2)}{e^2} \epsilon^{\mu\nu} \partial_\nu \square \phi), \quad (16)$$

and

$$\partial^\mu \partial_\nu A^\nu = -\frac{e}{m^2} \square \partial^\mu \phi. \quad (17)$$

Substituting the equations (16) and (17) into the equation (9) we find that the left hand side of the equation (9) turns into the following

$$\partial_\mu F^{\mu\nu} + e(g^{\mu\nu} - \epsilon^{\mu\nu}) \partial_\nu \phi + m^2 A^\mu = \square A^\mu - \partial^\mu \partial_\nu A^\nu + e(g^{\mu\nu} - \epsilon^{\mu\nu}) \partial_\nu \phi + m^2 A^\mu. \quad (18)$$

If we now substitute  $A_\mu$  in (18), the right hand side of the equation (18) turns into  $\frac{e^2 - m^2}{em^2} (\square + \frac{m^4}{m^2 - e^2}) \square(\phi + \chi)$ . So in order to satisfy equation (9) the field combination  $(\phi + \chi)$  must satisfy the equation

$$(\square + \frac{m^4}{m^2 - e^2}) \square(\phi + \chi) = 0. \quad (19)$$

So, it is clearly seen that with the definition  $A_\mu$  given in (13), the Euler-Lagrange equations (9) and (10) followed from the lagrangian (7), get satisfied when the following two conditions hold.

$$(\square + \frac{m^4}{m^2 - e^2}) \square(\phi + \chi) = 0, \quad (20)$$

$$\square \chi = 0. \quad (21)$$

In fact, these are the theoretical spectra in the Lagrangian formulation. The equation (20), describes a free massive boson with square of the mass  $\tilde{m}^2 = \frac{m^4}{m^2 - e^2}$  and equation (21) represents a massless boson. Therefore, the above computation over the proposed model in the lagrangian formulation suggests that the system under consideration contains two free fields  $\square(\phi + \chi)$  and  $\chi$ . It also indicates that one of these two fields  $\square(\phi + \chi)$  carries a mass with square of the mass  $\tilde{m}^2 = \frac{m^4}{m^2 - e^2}$  but the other one  $\chi$  is of vanishing mass.

Though the equations of the spectrum (20) and (21), come out as a condition to satisfy equation (9) and (10), it does not mean that these two do not have any physical significance. We will be able to see a meaningful correspondence of these two with the spectrum that would be obtained in the Hamiltonian formulation which we are going to consider in the next section. Before that let us see what does the combination of the fields  $(\phi + \chi)$  stands for. To see it let us compute the expression for  $\epsilon_{\mu\nu} \partial^\nu A^\mu$ . We find that in  $(1+1)$  dimension

$$\pi_1 = -\epsilon_{\mu\nu} \partial^\nu A^\mu = \frac{m^2 - e^2}{m^2 e} \square(\phi + \chi). \quad (22)$$

Here  $\pi_1 = \dot{A}_1 - A'_0$ , the momenta corresponding to the field  $A_1$ . The condition  $\square \chi = 0$ , is also required to get the relation (22). The equation (22) will be useful to see the correspondence between the spectra coming out here from lagrangian formulation to the spectrum that would come out from the hamiltonian formulation which we have described in the last paragraph of the next section. We will now proceed to carry out the hamiltonian formulation of the system.

#### IV. CONSTRAINT ANALYSIS AND DETERMINATION OF THE THEORETICAL SPECTRUM

To study the model in the hamiltonian formulation, the standard quantization of constrained system due to Dirac has been employed here. This formulation not only helps to get the theoretical spectrum but also it enables to understand the positive definiteness of the energy of this system during the course of its analysis. Needless to mention that positive definiteness is the first step of ensuring the unitary property of a field theoretical model. To investigate the fate of this model in the present scenario we thus proceed to quantize the theory.

Applications of Dirac's formalism towards the quantization of the proposed theory involves the identification of the constraints of the same that remains embedded within its phase space. We, therefore, proceed to identify the constraint of the theory at the early stage of our analysis. To this end, we require to calculate the canonical momenta of the fields with which the model is constituted. The momentum corresponding to the field  $\phi$ ,  $A_0$  and  $A_1$  respectively are

$$\pi_\phi = \dot{\phi} + e(A_0 - A_1), \quad (23)$$

$$\pi_0 = 0, \quad (24)$$

$$\pi_1 = \dot{A}_1 - A'_0. \quad (25)$$

The above three equations (23), (24) and (25), help us to obtain the hamiltonian through the Legendre transformation

$$H_B = \int dx [\pi_\phi \dot{\phi} + \pi_1 \dot{A}_1 + \pi_0 \dot{A}_0 - \mathcal{L}], \quad (26)$$

which ultimately gives the following hamiltonian density for the system we are interested in.

$$\mathcal{H}_B = \frac{1}{2}(\pi_1^2 + \pi_\phi^2 + \phi'^2) + \pi_1 A'_0 - e(\pi_\phi + \phi')(A_0 - A_1) + \frac{1}{2}e^2(A_0 - A_1)^2 - \frac{1}{2}m^2(A_0^2 - A_1^2). \quad (27)$$

Equation (24), is independent of  $\dot{A}_0$ . So it is the primary constraint of the theory. According to Dirac's prescription [27], the further analysis from this stage would have to be done using the effective hamiltonian instead of using the canonical hamiltonian obtained directly from the Legendre transformation in equation (27), and the effective hamiltonian in this situation is

$$H_{eff} = H + \int dx u_0 \pi_0. \quad (28)$$

The Lagrangian multiplier (velocity)  $u_0$  is yet to be determined. It will be fixed later. From the point of view of the physical consistency it stands as an essential requirement that the primary constraint is constrained to satisfy the condition  $\dot{\pi}_0 = [H(x), \pi_0(y)] \approx 0$ , because it has to be preserved for all time and that leads to the secondary constraint

$$G = \pi'_1 + 2e(\pi_\phi + \phi') + (m^2 - e^2)A_0 + e^2 A_1 \approx 0. \quad (29)$$

It is known as the Gauss law of the theory. This constraint (29), needs to preserve like the primary constraint (24). However, preservation of the constraint (29), does not give rise to any new constraint. Instead, it fixes the velocity  $u_0$ . This indicates that these two second class constraints are embedded within the phase space of the system and these two stand as the initial input to calculate the Dirac brackets.

According to the Dirac terminology [27], the constraints (24) and (29) both are weak conditions up to this stage. If it is now attempted to impose these into the hamiltonian treating these two as strong condition, the hamiltonian will be then be reduced to

$$\begin{aligned} H_R = & \int dx \left[ \frac{1}{2}\pi_1^2 + \frac{1}{2}\frac{1}{m^2 - e^2}\pi_1'^2 + \frac{1}{2}\frac{m^2}{m^2 - e^2}(\pi_\phi^2 + \phi'^2) + \frac{e^2}{m^2 - e^2}\pi'_1 A_1 \right. \\ & + \frac{e^2}{m^2 - e^2}\pi_\phi \phi' + \frac{e}{m^2 - e^2}\pi'_1 \phi' + \frac{e}{m^2 - e^2}\pi'_1 \phi' + \frac{em^2}{m^2 - e^2}A_1 \pi_\phi \\ & \left. + \frac{em^2}{m^2 - e^2}A_1 \phi' + \frac{1}{2}\frac{m^4}{m^2 - e^2}A_1^2 \right]. \end{aligned} \quad (30)$$

But the price that has to be paid for this is to replace the canonical Poission brackets by the corresponding Dirac bracket [27] because the Poission brackets become inadequate when the constraints are plugged in strongly into the Hamiltonian. It is known that Dirac bracket between the two variables  $A(x)$  and  $B(y)$  is defined by

$$[A(x), B(y)]^* = [A(x), B(y)] - \int [A(x)\omega_i(\eta)]C_{ij}^{-1}(\eta, z)[\omega_j(z), B(y)]d\eta dz, \quad (31)$$

where  $C_{ij}^{-1}(x, y)$  is given by

$$\int C_{ij}^{-1}(x, z)[\omega_i(z), \omega_j(y)]dz = 1. \quad (32)$$

Here  $\omega_i$ 's represents the second class constraints that remains embedded within the phase space of the theory. The matrix  $C^{-1}(x, y)$  for the present theory is given by

$$C^{-1}(x, y) = \frac{1}{m^2 - e^2} \begin{pmatrix} 0 & \delta(x - y) \\ -\delta(x - y) & 0 \end{pmatrix}, \quad (33)$$

With the help of equation (31), the Dirac brackets among the fields  $A_1$ ,  $\pi_1$ ,  $\phi$ , and  $\pi_\phi$  are calculated:

$$[A_1(x), A_1(y)]^* = 0 \quad (34)$$

$$[A_1(x), \pi_1(y)]^* = \delta(x - y) \quad (35)$$

$$[\phi(x), \phi(y)]^* = \delta(x - y) \quad (36)$$

$$[\phi(x), \pi_\phi(y)]^* = 0 \quad (37)$$

Note that the above equations imply that the Dirac brackets retain its own Poission bracket structures here which was not the case when it was studied in [8] with the Faddeevian class of regularization. Making use of the Dirac brackets (34), (35), (36) and (37), the equations of motion for the fields with which the hamiltonian (30) is constituted are computed and it is found that the following first order equations of motion are resulted in

$$\dot{A}_1 = \pi_1 - \frac{e^2}{m^2 - e^2}A_1' - \frac{e}{m^2 - e^2}\pi_\phi' - \frac{e}{m^2 - e^2}\phi'' - \frac{1}{m^2 - e^2}\pi_1'' \quad (38)$$

$$\dot{\pi}_1 = -\frac{e^2}{m^2 - e^2}\pi_1' - \frac{em^2}{m^2 - e^2}\pi_\phi - \frac{em^2}{m^2 - e^2}\phi' - \frac{m^4}{m^2 - e^2}A_1 \quad (39)$$

$$\dot{\phi} = \frac{m^2}{m^2 - e^2}\pi_\phi + \frac{e^2}{m^2 - e^2}\phi' + \frac{e}{m^2 - e^2}\pi_1' + \frac{em^2}{m^2 - e^2}A_1 \quad (40)$$

$$\dot{\pi}_\phi = \frac{m^2}{m^2 - e^2}\phi'' + \frac{e^2}{m^2 - e^2}\pi_\phi' + \frac{e}{m^2 - e^2}\pi_1'' + \frac{em^2}{m^2 - e^2}A_1' \quad (41)$$

The above first order equations of motion get simplified into the following second order differential equations after a little algebra.

$$(\square + \frac{m^4}{m^2 - e^2})\pi_1 = 0, \quad (42)$$

$$(\square + \frac{m^4}{m^2 - e^2})(A_1 + \frac{e}{m^2}\phi) = 0, \quad (43)$$

$$\square(\phi + \frac{e}{m^2}\pi_1) = 0, \quad (44)$$

$$\square(\pi_\phi + \frac{e}{m^2}\pi'_1) = 0 \quad (45)$$

The above four equations (42), (43), (44) and (45) suggest that the field  $A_1 + \frac{e}{m^2}\phi$  describe a massive boson with square of the mass  $\tilde{m}^2 = \frac{m^4}{m^2 - e^2}$  and the field  $\phi + \frac{e}{m^2}\pi_1$  represents a boson with vanishing mass. The field  $\pi_1$  and  $\pi_\phi + \frac{e}{m^2}\pi'_1$  may be considered as the momenta corresponding to the field  $A_1 + \frac{e}{m^2}\phi$  and  $\pi_\phi + \frac{e}{m^2}\pi'_1$  respectively because the pair of fields describing equations 42) and (43) satisfy canonical poisson bracket among themselves and the pairs describing the equations (44) and (45) also satisfy the canonical poisson bracket. Note that  $m^2$  must be greater than  $e^2$  in order to get the mass of the massive boson a physical (positive) one. The condition  $m^2 > e^2$  is essential in order the Hamiltonian (30) to be positive definite which is the basic footing of ensuring uniatrity of the theory. One more thing which we would like to address here is that some practical benefit may also be followed from this model. The massless bosons found in the spectrum is equivalent to a massless fermion in two dimensions so it can be taught of as fermion in a de-confined state [14, 20, 21, 24].

We will now be able to see the correspondence between the spectrum obtained here and the spectrum that came out in lagrangian formulation. If we look carefully towards the equations (20), (22) and (42) we will be able to see that the equations (20) and (42) are identical. Using equation (22) and (42) it can be ensured that the massless field  $\chi$  that appeared in the lagrangian formulation is nothing but the field  $\phi + \frac{e}{m^2}\pi_1$ . So the spectrum that came out as a condition in the lagrangian formulation does not fail to carry proper physical meaning.

## V. INCLUSION OF COUNTER TERM CONTAINING THE AMBIGUITY PARAMETERS

In Sec. II, Chiral Thirring-Wess model has been defined by the following generating functional

$$Z[A] = \int d\psi d\bar{\psi} e^{\int d^2x \mathcal{L}_f} \quad (46)$$

where the definition of  $\mathcal{L}_f$  is available from the equation (2). As mentioned in Sec. II, the dynamics of the  $A_\mu$  field is governed by he Proca field and the lagrangian of which is given in equation (3) In Sec. II, it has already been mentioned that in the lagrangian (2), right handed fermion remains uncoupled when vector interaction is replaced by the chiral interaction. So integration over this right handed part leads to field independent counter part which can be absorbed within the normalization. However the integration over the left handed fermion is much involved because one needs to regularize the fermionic determinant during the process of integration since the determinant has a diverging nature, and after a careful calculation one arrives at the following generating functional [8, 20, 22, 23]

$$\begin{aligned} Z[A] &= \int d\psi_L d\bar{\psi}_L \bar{\psi}_L \gamma^\mu (i\partial_\mu + 2e\sqrt{\pi}A_\mu)\psi_L \\ &= \exp\frac{ie^2}{2} \int d^2x A_\mu [M_{\mu\nu} - (\partial^\mu + \tilde{\partial}^\mu) \frac{1}{\square} (\partial^\nu + \tilde{\partial}^\nu)] A_\nu, \end{aligned} \quad (47)$$

In general, the elements of the  $M_{\mu\nu}$  can take any arbitrary values. However, the model looses both its solvability and Lorentz invariance in that situation [8]. In [8], we considered a symmetric form of  $M_{\mu\nu}$ :

$$M_{\mu\nu} = \begin{pmatrix} \tilde{a} & \alpha \\ \alpha & \gamma \end{pmatrix} \delta(x - y). \quad (48)$$

where regularization ambiguity got involved within the parameters  $\tilde{a}$ ,  $\alpha$  and  $\gamma$ . These parameters entered there in order to remove the divergence of the fermionic determinant since the evaluation of the determinant needs a *one loop correction* [20, 22, 23]. It was found in [8] that all the parameters did not remain independent with each other. Some constrained among the ambiguity parameters were found to be essential for the model to be physically sensible.

This generating functional (4), when written there in terms of the auxiliary field  $\phi(x)$  it turned out to the following

$$Z[A] = \int d\phi e^{i \int d^2x \mathcal{L}_B}, \quad (49)$$

with

$$\mathcal{L}_B = \frac{1}{2}(\dot{\phi}^2 - \phi'^2) + e(\dot{\phi} + \phi')(A_0 - A_1) + \frac{1}{2}e^2(\tilde{a}A_0^2 + 2\alpha A_0 A_1 + \gamma A_1^2). \quad (50)$$

So the total lagrangian density with which we dealt there was

$$\mathcal{L} = \frac{1}{2}(\dot{\phi}^2 - \phi'^2) + e(\dot{\phi} + \phi')(A_0 - A_1) + \frac{1}{2}(\dot{A}_1^2 - A_0'^2) + \mathcal{L}_{mass} \quad (51)$$

where the term  $\mathcal{L}_{mass}$  was

$$\mathcal{L}_{mass} = \frac{1}{2}e^2[(\tilde{a} + \frac{m^2}{e^2})A_0^2 + 2\alpha A_0 A_1 + (\gamma - \frac{m^2}{e^2})A_1^2]. \quad (52)$$

This Lagrangian in general failed to provide Poincaré invariant equations of motion. Ambiguity in the regularization allowed us to set two conditions  $\tilde{a} + \frac{m^2}{e^2} = 1$  and  $m^2 = e^2(1 + \gamma - 2\alpha)$  without violating any physical principle. In our previous work [8], we showed that the condition  $\tilde{a} + \frac{m^2}{e^2} = 1$  helped us to fit the model within the Faddeevian class [28–31] and the theory rendered an interesting Lorentz invariant theoretical spectrum though there was no Lorentz covariance in the starting lagrangian provided the constraint  $m^2 = e^2(1 + \gamma - 2\alpha)$  among the ambiguity parameters were maintained. Indeed, some other choices may also lead to some other physically sensible theories. We are free to choose Jjackiw-Rajaraman type of regularization for this model. In that case we have to choose the matrix  $M_{\mu\nu}$  as  $M_{\mu\nu} = ag_{\mu\nu}$ . It keeps the model Lorentz covariant to start with and the lagrangian density for this situation reads

$$\mathcal{L} = \frac{1}{2}(\dot{\phi}^2 - \phi'^2) + e(\dot{\phi} + \phi')(A_0 - A_1) + \frac{1}{2}(\dot{A}_1^2 - A_0'^2) + \frac{1}{2}e^2(a + \frac{m^2}{e^2})(A_0^2 - A_1^2) \quad (53)$$

Note that the only difference between the model given in equation (53) and the model considered in equation (7), lies in the masslike terms for gauge fields. To be more precise the difference between ambiguity (one loop correction) free situation and the situation where ambiguity (one loop correction) is taken into consideration lies in the masslike terms for gauge fields. The masslike term in the lagrangian (53) now turns into  $\frac{1}{2}e^2(a + \frac{m^2}{e^2})(A_0^2 - A_1^2)$ . Note that the constant  $m^2$  of the lagrangian (7) is now shifted to  $e^2(a + \frac{m^2}{e^2})$ . All the other terms remain unaltered. So the analysis will also follow the same direction as it has been made in Sec. II. and Sec. III. Unlike the situation studied in [8], the constraint structure will also remain the same except the shifting of the constant parameter  $m^2$  to  $e^2(a + \frac{m^2}{e^2})$ . A careful look will help to understand without much difficulty that there will be a shift in the mass of the massive field because of this shifting of the constant parameter  $m^2$  in the masslike term of (53). Without going through the detail analysis it can be inferred that the square of the mass of the massive boson that will follow from the lagrangian (53) will be none other than  $\tilde{m}^2 = \frac{m^4 + a^2 e^4 + 2am^2 e^2}{m^2 + e^2(a-1)}$ . However, the massless excitation will remain unaffected and which can be thought of as a de-confined fermion in  $(1+1)$  dimension as usual.

## VI. CONCLUSION

In this paper we have studied the Thirring-Wess model replacing its vector type interaction by the chiral type. The model has been analyzed both in the lagrangian and hamiltonian formulation. Drawing Euler-Lagrange equations into service we have obtained that the model bears a rich Lorentz invariant theoretical spectrum. The use of the standard method of quantization of constrained system by Dirac [27] too confirms the same in much more transparent manner. The spectrum contains a massive boson like the usual vector Schwinger model. A massless boson has also been found to appear like the chiral Schwinger model. One more information which the hamiltonian of the present model provides for is that its positive definiteness is maintained for the restriction  $m^2 > e^2$ . It is indeed the basic requirement of a theory to be sound in true physical sense. Not only that but also it is the first step of ensuring the unitarity of the theory. The model however does not possess gauge symmetry. It is not unnatural since its ancestor, i.e., Thirring-Wess model is not gauge symmetric like the vector Schwinger model.

The point on which we would like to emphasize is that the chiral generation studied here does not need any counter term in order to make it physically sensible like the chiral generation of the Schwinger model made in [13]. It is found from our analysis that unlike the chiral generation of Schwinger model [13] this model does not fail to maintain unitarity and exactly solvable nature in the absence of any counter term containing ambiguity parameter, and as a result, no ambiguity parameter entered into the mass of the massive boson. In this sense, Chiral generation of the Thirring-Wess model scores over the chiral generation of Schwinger model [13] since it carries less complication, and this would not suffer from any un-physical situation like the chiral generation of Schwinger model [13].

It has already been mentioned that the gauge symmetry is absent here like its ancestor. Though the absence of gauge symmetry does not violate any physical principle nevertheless one may argue that a gauge symmetric theory is advantageous because it reflects an increased symmetry of the lagrangian, albeit it has to be remembered that what is



increased is not a physical symmetry of the states but only a symmetry of the effective action which has to be broken by gauge fixing. Even though, if we allow some respect or emphasis on the gauge symmetry of the effective action that can also be met here with out much difficulty. There is certainly room for converting this gauge non-invariant model into a structure where gauge symmetry is maintained and that too within the ambit of its own physical phase space. The method of converting a gauge non-invariant model into a gauge invariant one as suggested by of Mitra and Rajaraman [32, 33] may be useful in this context. The method of bringing back gauge symmetry of a gauge non-invariant model in the extend phase spacer is also well known and can be applied here as well. However, one needs to introduce the appropriate Wess-Zumino term [34] in that case. In that situation some extra fields will enter within the effective action. But it is not a matter to get worried because the extra fields allocate themselves within the un-physical sector of the theory.

The hamiltonian of the model under present consideration is positive definite that gives the signature of maintenance of unitarity. However, the the formal proof of certainly comes from the BRST quantization and there is no difficulty to carry it out for this model since it provides positive definite hamiltonian. Therefore, this model is able to satisfy all the important criteria in order to be granted as physically sensible in the lower dimensional field theoretical regime from whatever aspects it have been looked for.

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